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Hungarian Mechanism based Sectored FFR for Irregular Geometry Multicellular Networks

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Abstract

The growing demands for mobile broadband application services along with the scarcity of the spectrum have triggered the dense utilization of frequency resources in cellular networks. The capacity demands are coped accordingly, however at the detriment of added inter-cell interference (ICI). Fractional Frequency Reuse (FFR) is an effective ICI mitigation approach when adopted in realistic irregular geometry cellular networks. However, in the literature optimized spectrum resources for the individual users are not considered. In this paper Hungarian Mechanism based Sectored Fractional Frequency Reuse (HMS-FFR) scheme is proposed, where the sub-carriers present in the dynamically partitioned spectrum are optimally allocated to each user. Simulation results revealed that the proposed HMS-FFR scheme enhances the system performance in terms of achievable throughput, average sum rate, and achievable throughput with respect to load while considering full traffic.

Keywords: ICI, FFR, Irregular Geometry, Hungarian Mechanism

1. INTRODUCTION

Recent drifts in mobile broadband application services have resulted in a desperate growth in data traffic. Consequently, challenging the network operators to considerably increase their system capacity. In literature, this challenge is tackled by reusing all the spectrum resources at each cell of the network. System capacity is boosted accordingly, however, at the expense of ICI because of the co-channel operation in the neighboring cells [1].

To address this issue, static intercell interference coordination (ICIC) methods have been adopted which include FFR [2]. FFR merges the advantages of low and high-frequency reuse. This is done by partitioning the cell coverage into cell-center and cell-edge sub-regions [3]. Moreover, the frequency spectrum is divided, one part is reused with factor one for all cell-center users, whereas the other part is used at the cell edge region with a high reuse factor. The low complexity of the FFR technique makes it an attractive interference mitigation approach along with its ability of considerable coverage enhancement. However, the performance of static FFR scheme is not remarkable with the realistic cellular deployment. Accordingly, sectored-FFR schemes are proposed[4], where 120-degree sectoring is implemented to further divide the cell edge region. Moreover, in sectored-FFR, the second part of the spectrum for the cell edge region is utilized with a reuse factor of one. Therefore, resulting in full spectrum utilization at each cell of the network.

The rest of the paper is organized as the related work is presented in section 2, section 3 gives the system model design along with the formulation of the optimization problem; section 4 presents the development of the proposed HMS-FFR scheme. Performance analysis is performed in section 5, whereas section 6 concludes the paper.

2. RELATED WORKS

Network geometry or topology deliberated for ICI mitigation scheme plays a vital role in the performance evaluation of cellular systems [5]. In a realistic deployment with an irregular cellular layout, the signal propagation conditions are independent for each cell and hence experience very different amounts of ICI [6]. Since the signal to noise plus interference ratio is greatly affected by the position of base stations (BSs) and the network users. Therefore, the performance of traditional FFR schemes designed for regular geometry hexagon models [7] is inadequate in the realistic irregular geometry cellular deployment [8].

These aspects have prompted the research and FFR has been considered with irregular geometry cellular networks [9]. However, previous research which encounters FFR into irregular geometry models is mostly the simplest version of FFR where only two regions (cell-center and cell-edge) are considered while ignoring cell sectoring. Moreover, dynamic spectrum utilization with FFR configuration is required to be optimized when applied to the irregular geometry cellular network [10]. The dynamic spectrum allocation in FFR enables the available spectrum to be divided following the network variants and miscellaneous traffic demands [11]–[13].

In our previous work [14], a Sectored-FFR scheme has been developed for the ICI mitigation in irregular geometry OFDMA multicellular network. The spectrum resources are fully utilized at each cell of the network by dividing the cells into a number of sub-regions. The available frequency spectrum is dynamically apportioned into sub-bands, one for each sub-region, according to the traffic demand or load condition. However, the optimized bandwidth resources or the number of sub-carriers for the individual users are not considered. In this work, the HMS-FFR scheme is proposed as an enhancement of the Sectored-FFR scheme, to optimally allocate the spectrum resources among the cooperative users. The proposed HMS-FFR is based on the Hungarian algorithm [15], with the ability to optimally assign the available sub-carriers to individual users according to their channel conditions.

The Hungarian method was formulated by Harold W. Kuhn in 1955 [15] and was designed to solve the assignment problem. The assignment problem is a combinatorial optimization problem in operational research. The basic form of the assignment problem is stated as there are several agents and tasks. Each agent can accomplish one task for a certain cost; the cost is not fixed and vary from each combination of task and agent. The objective of the assignment problem is to assign each agent exactly one task in such a way that the total cost of the assignment is minimized. In case, the number of tasks and agents are the same, the assignment problem is called the linear assignment problem.

To solve the linear assignment problem, the Hungarian algorithm modeled the tasks and agent in an (NxN) matrix, called the cost matrix. The element of the matrix is the cost of performing certain tasks, where each row represents a worker and each column represents a certain task. Hungarian algorithm is adopted to determine the lowest cost. Therefore, the original Hungarian algorithm was developed to solve the minimization problem. Following are the steps of the Hungarian algorithm for a minimization problem;

1. Determine the lowest value of each row and subtract it from every value of the row.
2. Determine the lowest value of each column and subtract it from every value of the column.
3. Use the minimum number of lines (horizontal and vertical) to cover all the zeros in the matrix
   a. If the number of lines is equal to the size of the matrix, the solution is possible.
   b. If the number of lines is less than the size of the matrix, then determine the minimum uncovered value, subtract it from all uncovered rows and add it to all uncovered columns.
4. Repeat step 3, until a solution is found.

Hungarian mechanisms can be applied to both maximization and minimization problems. Moreover, the Hungarian algorithm can be used with both balanced (Rows = Columns) or unbalanced (Rows ≠Columns) allocation problems.

3. SYSTEM MODEL DESIGN

In the system model, OFMDA based multi-cellular network is considered where fixed powered (P^t) BSs are operational in the downlink. To apprehend the realistic network deployment, BSs are considered to be spatially distributed corresponding to Hard Core Point Process (HCPP) in the Euclidean plane [16]. Furthermore, it is assumed that mobile users are randomly distributed in the coverage area and are connected to its nearest BS. The Voronoi tessellations characterize the cell boundaries or a BS coverage area [17]. Moreover, we assume that a user can register with only one BS, hence, the system operation denies multiple BSs access for a single user. getting network access by more than one BS is denied by the system operation. Moreover, user traffic demand is considered using the full buffer model.

Specifically, the minimum data traffic demands for individual users are randomly generated within a certain limit of data rate. Therefore, during the simulation, there are always data to be transmitted by the specific number of users in the network.

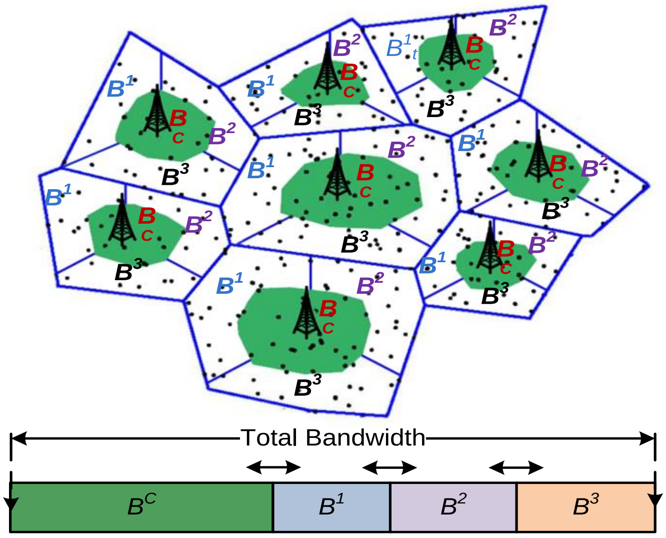

Figure 1. System Model

To analyze the execution of the proposed network topology, it is critical to accurately model the effect of the radio propagation channel on the received signal. The downlink received $SINR$ of a user n on sub-carrier k can be figured for a serving BS i is given in the following equation as derived in [14].

$$SINR_{n,k} = \frac{P^i_{n,k} G^i_{n,k}}{N_o \Delta f + \sum_{J \in \Phi \setminus i} P^j_{n,k}\, G^j_{n,k}} \tag{1}$$

Where $P^i_{n,k}$ is the received power on the sub-carrier $k \in K$ by user n, $G^i_{n,k}$is the channel gain of the BS i on the sub-carrier k for user n, N_o is the power spectral density of the additive white Gaussian noise (AWGN), Φ represents all transmitting BSs. The transmitted power and gain of the interfering BSs are represented by $P^j_{n,k}$ and $G^j_{n,k}$ respectively. The average received $SINR$ is considered as a threshold (T_S) to group the users into cell-center and cell-edge users. The BS classify the users N based on T_S, users with the $SINR$ values greater than T_S are marked as cell-center users N^C, whereas, users with average $SINR$ less that T_S are marked as cell-edge users N^E. Cell partition based on T_S is shown in Figure. 1, where users located in the green region are cell-center users. In addition to the cell partition, the cell-edge region is further divided into three sectors (120° Sectoring). Since the cells are of irregular geometry sectoring results in irregular sub-regions both in terms of geometry and traffic load. Hence, the spectrum resources are needed to be optimally allocated.

3.1 Problem Formulation

The users are of each sub-region are allowed to compete only for the sub-carriers in the sub-band specified for that sub-region. The total number of users $\mathcal{N}$ in the cell is the super-set of the users in each sub-region, that is

$$\mathcal{N} = \{N^c, N^1, N^2, N^3\} \quad (1)$$

Where, N^c represents the cell-center users. Similarly, N^1, N^2 and N^3 represent the number of users located in sector-1, sector-2, and sector-3, respectively. The total spectrum is having $\mathcal{K}$ number of sub-carriers,

$$\mathcal{K} = \{K^c, K^1, K^2, K^3\} \quad (2)$$

Where, K^c indicates the sub-carriers of the cell-center region sub-band, similarly, K^1, K^2 and K^3 are the number of sub-carriers in the sub-bands specified for sector-1, sector-2, and sector-3 of the cell-edge region, respectively.

Since OFDMA based downlink access is considered, the sub-carriers fade independently for each user in the cell, therefore, each user has different channel conditions (SINR values) for different sub-carriers. Moreover, each user may require more than one sub-carrier to achieve the target data rate R_n^t. The target data rate R_n^t is also different for each user while considering heterogeneous traffic demand. The throughput $R_{n,k}$ of a single user n on the sub-carrier k can be computed through Shanon's capacity formula, as

$$R_{n,k} = \Delta f \left(log_2\big(1 + SINR_{n,k}\big)\right) \quad (3)$$

Therefore, when a user n is assigned with a set of sub-carriers $K_n = \{1,2 \ldots K\}$, the R_n of a user can be calculated as

$$R_n = \sum_{k=1}^{K} \big(R_{n,k}\big) \quad (4)$$

The achievable throughput R_n of a user depends on the number of allocated sub-carriers and their SINR values. Therefore, the objective of the sub-carriers allocation is to maximize the achievable throughput of all users and hence the overall throughput of the cell. Consequently, the problem of the sub-carrier allocation can be formulated as the maximization problem. Mathematically the sub-carriers allocation problem can be expressed as

$$\underset{\{K \in (\,K^c, K^1, K^2, K^3)\}}{max} \left(\sum_{n=1}^{N} \sum_{k=1}^{K} x_{n,k}\, R_{n,k} \right) \quad (5)$$

$$\text{subject to}\begin{cases} x_{n,k} = \{0,1\} & \text{(5a)} \\ \sum_{n=1}^{N} x_{n,k} = 1, for\ \forall n & \text{(5b)} \\ \sum_{n=1}^{N^c} K_n \subseteq K^c & \text{(5c)} \\ \sum_{n=1}^{N^{(1,2,3)}} K_n \subseteq K^{(1,2,3)} & \text{(5d)} \end{cases}$$

Where $K^c, K^1, K^2, K^3$are the number of sub-carriers in the cell-center, sector-1, sector-2, and sector-3 sub-bands respectively. The constraint in (5a) indicates the problem is the binary integer type and (5b) indicates that each sub-carrier can be allocated to only one user in the cell. The constraint in the (5c) and (5d) ensures that users of any sub-region can request for a bundle of sub-carriers from the set of sub-carriers specified for that particular sub-region only. That is, the users in the set N_c can only get the sub-carriers (K_n) from the set of sub-carriers K_c. Similarly, users in the N_1 from the sub-carriers in the set K_1 , N_2 from the sub-carriers in K_2 and the set N_3 from the sub-carriers in the set K_3 respectively.

4. ORIGINALITY: PROPOSED HMS-FFR SCHEME

This section presents the development of the HMS-FFR scheme to solve the sub-carrier allocation problem given by Equation (5). The objective is of the HMS-FFR is to optimally allocate each sub-carriers to the user which has the highest SINR values for that sub-carriers. The detailed flow chart of the HMS-FFR algorithm is presented in Figure 2.

4.1 Development of Channel Matrices

The HMS-FFR algorithm operates on the frame level in a decentralized manner at each BS. First, at the initialization stage, the number of users (N) and the number of sub-carriers (K) of the sub-band specified for each sub-region of the cell are determined. The users are assumed to be cooperative to report the true channel state information (CSI) or the SINR information to the BS. The normalized values of the received SINR are used to generate the channel matrices ($H^C_{(N,K)}, H^1_{(N,K)}, H^2_{(N,K)}, H^3_{(N,K)}$) of each sub-region.

$$H^C_{(N,K)} = \begin{pmatrix} h^c_{1,1} & h^c_{1,2} & . & . & h^c_{1,k} \\ h^c_{2,1} & h^c_{2,2} & . & . & h^c_{2,k} \\ . & . & . & . & . \\ . & . & . & . & . \\ h^c_{n,1} & h^c_{n,2} & . & . & h^c_{n,k} \end{pmatrix} \quad (6)$$

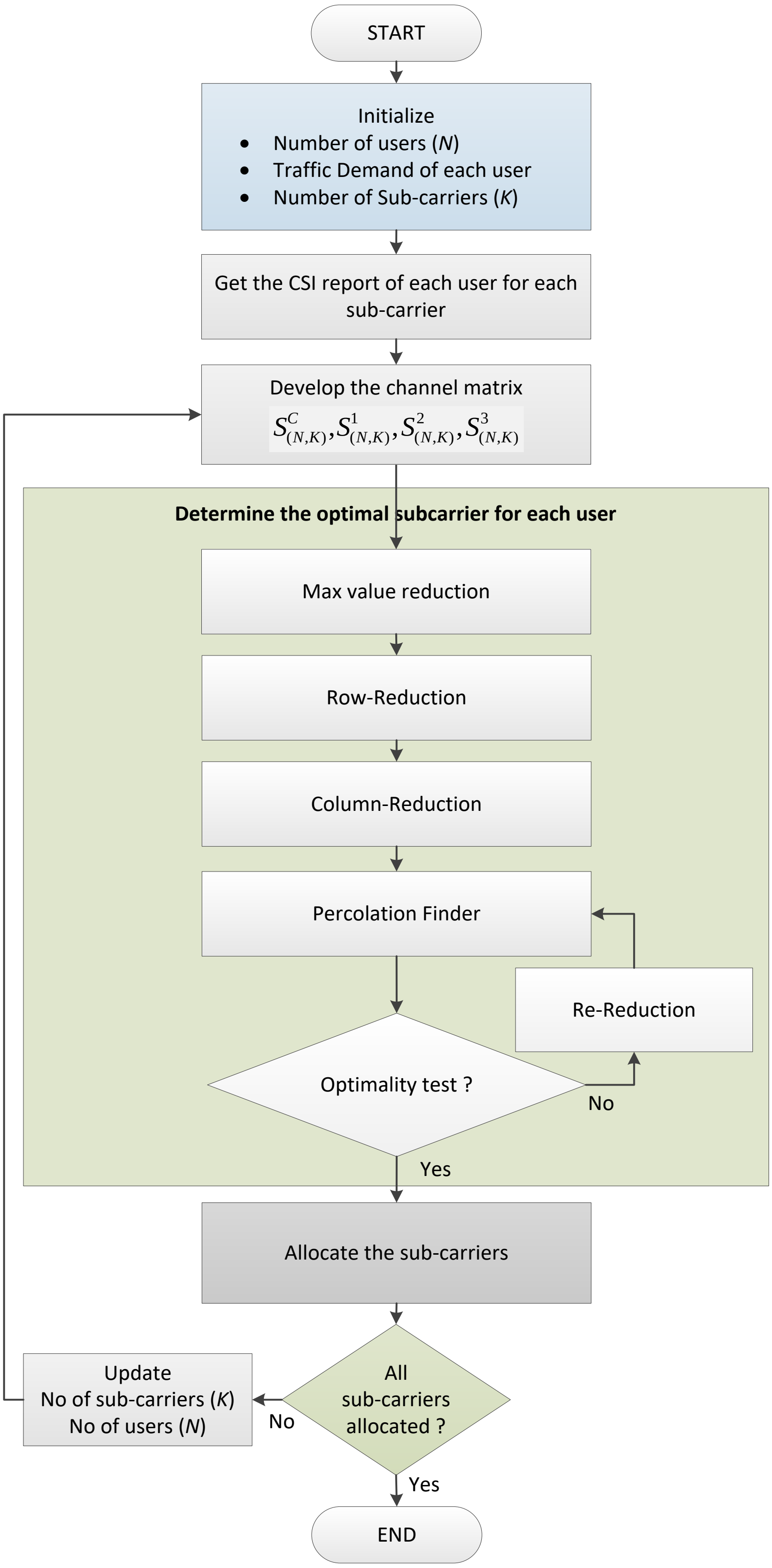

Figure 2. Flow chart of HMS-FFR Algorithm

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Similarly, for sector1, sector2, and sector3, the corresponding SINR metrics are given as

$$H^1_{(N,K)} = \begin{pmatrix} h^1_{1,1} & h^1_{1,2} & \cdot & \cdot & h^1_{1,k} \\ h^1_{2,1} & h^1_{2,2} & \cdot & \cdot & h^1_{2,k} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ h^1_{n,1} & h^1_{n,2} & \cdot & \cdot & h^1_{n,k} \end{pmatrix} \tag{7}$$

$$H^2_{(N,K)} = \begin{pmatrix} h^2_{1,1} & h^2_{1,2} & \cdot & \cdot & h^2_{1,k} \\ h^2_{2,1} & h^2_{2,2} & \cdot & \cdot & h^2_{2,k} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ h^2_{n,1} & h^2_{n,2} & \cdot & \cdot & h^2_{n,k} \end{pmatrix} \tag{8}$$

$$H^3_{(N,K)} = \begin{pmatrix} h^3_{1,1} & h^3_{1,2} & \cdot & \cdot & h^3_{1,k} \\ h^3_{2,1} & h^3_{2,2} & \cdot & \cdot & h^3_{2,k} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ h^3_{n,1} & h^3_{n,2} & \cdot & \cdot & h^3_{n,k} \end{pmatrix} \tag{9}$$

In Equation (6-9), each row of the matrix represents one user and each column represents one sub-carrier. The sub-carrier fades independently, therefore, different SINR values for different users. Since the number of the sub-carriers is more than the number of users, the generated matrices are not square, whereas the Hungarian method can only operate on the square matrix. Therefore, extra zeros are added to each matrix to make it a square matrix.

4.2 Optimal Sub-carrier Allocation

Once the channel matrices are formed, the proposed HMS-FFR algorithm executes the following steps to determine the optimal sub-carrier allocation for each user of each sub-region.

1. **Maximum Value Reduction**
   determine the maximum value of the matrix
   Subtract each value of the matrices from the maximum value
2. **Row Reduction**
   Determine the minimum value of each row of the resultant matrices of step 1.
   Subtract that minimum value from each value of the corresponding row.
   Each row will have at least one zero.

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3. Column Reduction
   Determine the minimum value of each column of the resultant matrices of step 2.
   Subtract that minimum value from each value of the corresponding column.
   Each column will also have at least one zero.
4. Percolation
   Draw a minimum number of the horizontal and vertical lines so that all the zeros covered
5. Optimality Test
   If the number of lines is equal to the number of users N
           Optimal sub-carrier allocation is possible
           Allocate the sub-carrier corresponds to zero entries in the original matrix
   Else
           Optimal sub-carrier allocation is not possible
           Go to the next step
   End if
6. Re-Reduction and addition
   Determine the maximum value which is not covered by any line.
   Subtract this value from each uncovered row and add it to each covered column.
   Go back to step 4.
```

After each allocation, the HMS-FFR scheme checks the remaining available sub-carriers. If there are still sub-carriers to allocate, the number of sub-carriers K and number of users N are updated accordingly. The number of users is updated because if the demand of the users is fulfilled, they are eliminated from the users set which can get the remaining sub-carriers. The channel matrices are formed again based on the CSI reports, and the process continues until all the sub-carriers are allocated.

5. PERFORMANCE ANALYSIS

This section gives the performance analysis of the proposed HMS-FFR scheme for the irregular geometry cellular network for the same simulation setup as presented in [14]. To make the analysis more significant and meaningful, the proposed schemes are compared with basic FFR-3, and Dynamic-FFR3 [14] schemes respectively. In the basic FFR-3 scheme the total available spectrum is divided into a number of sub-bands of fixed size and are allocated to each sub-region of the cell. In the Dynamic-FFR3 scheme, the sub-band for each sub-region is calculated according to the traffic demand of each sub-region. Then the sub-carriers of each sub-band are randomly allocated to the users of that particular sub-region. Whereas, in the proposed scheme the sub-carriers are optimally allocated to the users as described in the previous section.

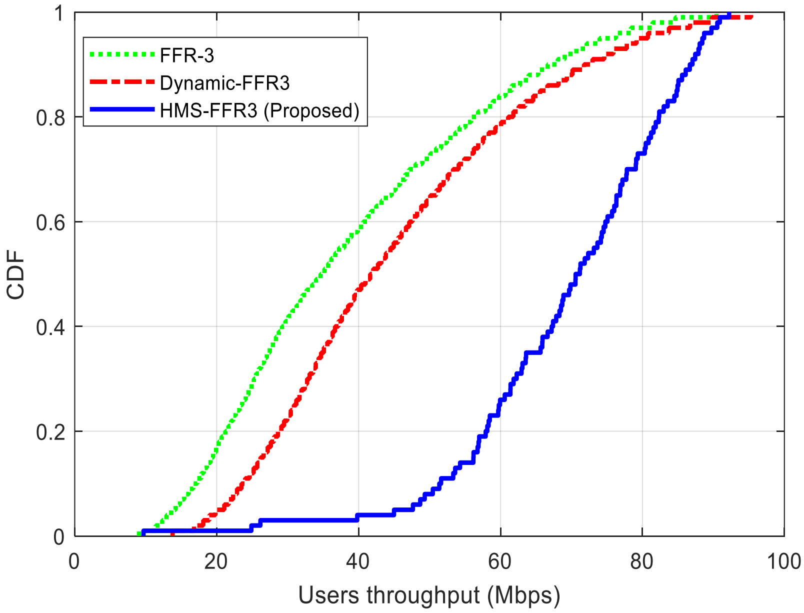

Figure 3. CDF of the users' throughput

5.1 Achievable Throughput

The achievable throughput is computed based on Shannon's theorem, which leads to the upper bound of the user throughput in bits per second. The CDF of the user's throughput is shown in Figure 3. The result shows that compared to basic FFR-3 and Dynamic-FFR3, users' achievable throughput significantly increases with the proposed optimal sub-carriers allocation through the HMS-FFR3 scheme. Specifically, compared to the Dynamic-FFR3 scheme, 53.2% improvement in the users' achievable throughput is recorded for HMS-FFR. Compared to the basic FFR-3 scheme, a 72.3% increase in the users' throughput is achieved through HMS-FFR. The worst performance of the basic FFR-3 in terms of achievable throughput is because of the static sub-band allocation to each sub-region of the cell. HMS-FFR3 exploits the multiuser diversity and allows users to report the true CSI and accordingly the sub-carriers are optimally allocated.

5.2 Average sum rate

The average sum rate of the cell, cell-center, and cell-edge region for HMS-FFR3, Dynamic-FFR3, and FFR-3 schemes are plotted in Figure.4. The average sum rate plot shows that the HMS-FFR3 improves the average sum rate of the cell by 53.1% and 75.2% compared to the Dynamic-FFR3 and FFR-3 schemes respectively.

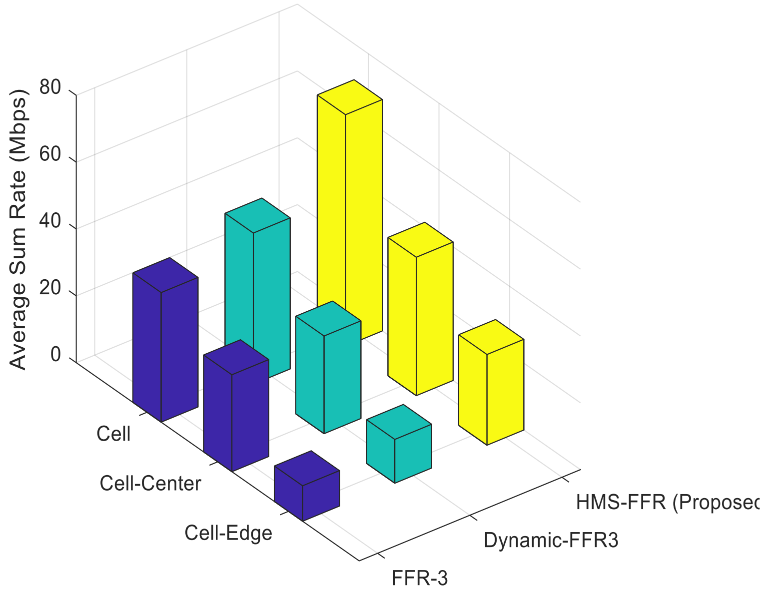

Figure 4. Average sum rate

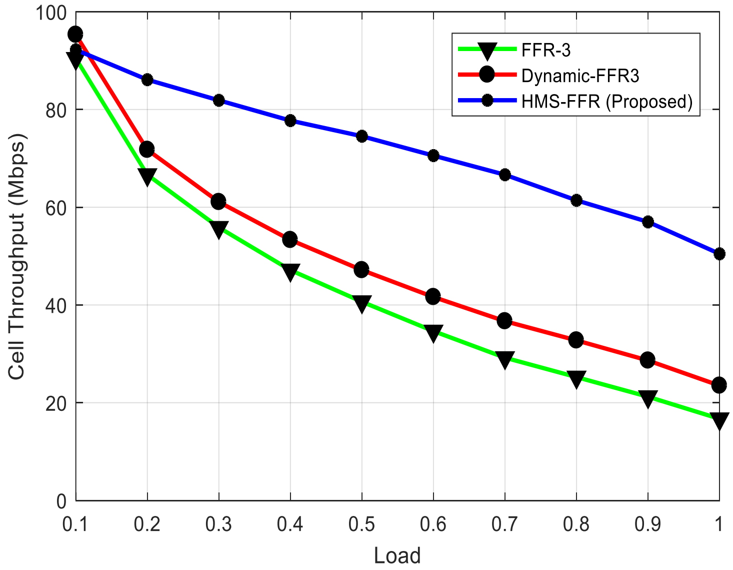

Figure 5. Achievable throughput with respect to load

5.3 Achievable Throughput to load

Since we have considered heterogeneous traffic demand by the users of each cell, which is translated to the load as given by equation (47) in [14]. This section analyzes the performance of the proposed AMS-FFR3 in terms of achievable throughput for the load distribution. The results for the cell achievable throughput with respect to load is shown in the Figure. 5. When the value of the load approaches 1, implies that 100% load of the cell. AMS-FFR3 scheme outperforms the Dynamic-FFR3 and FFR-3 schemes by 48% and 65.3% respectively.

6. CONCLUSION

In this paper, the HMS-FFR scheme is developed as an ICI mitigation scheme for a realistic irregular geometry multicellular network. Since the geometry of the cells is irregular, cell-partition and sectoring result in the sub-regions of different coverage areas and hence different numbers of users. The proposed HMS-FFR is based on the Hungarian method to optimally allocate the sub-carriers to users in each sub-region. HMS-FFR determines the channel matrix for each sub-region of the cell-based CSI reports received from users. Then the sub-carriers are allocated to the user with maximum SINR values. The simulation results show that the proposed HMS-FFR scheme improves the users' achievable throughput by 72.3% and 53.2% compared to basic FFR-3 and dynamic-FFR schemes respectively. Results for the average sum throughput show that HMS-FFR improves it by 75.2% and 53.1% compared to basic FFR-3 and dynamic-FFR schemes respectively. Similarly, the achievable throughput with respect to load is improved by 65.3% and 48% compared to basic-FFR3 and dynamic-FFR schemes respectively. The Fifth-Generation (5G) cellular systems and beyond are aiming full spectrum reuse (Reuse-1) in each cell of the network along with heterogeneous multi-tier deployment. Therefore, to fully utilize the available spectrum in each cell of the network, the proposed HMS-FFR scheme may be extended for optimal resource allocation in heterogeneous multi-tier deployment and as a potential interference mitigation scheme in the future 5G cellular networks.

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